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# **A Neo-Kaldorian Approach to Structural Economic Dynamics**

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## **Abstract**

Although the Structural Economic Dynamic approach provides a simultaneous consideration of demand and supply sides of economic growth, it does not fully take into account the possible role played by demand in the generation of technical progress. From a neo-Kaldorian perspective, this paper seeks to establish the concepts of demand and productivity regimes in an open version of the pure labour Pasinettian model. In order to derive the demand regime, a disaggregated version of the static Harrod foreign multiplier is derived, while the productivity regime is built in terms of disaggregated Kaldor-Verdoorn laws. The upshot is a multi-sectoral growth model of structural change and cumulative causation, in which an open version of the Pasinettian model to foreign trade may be obtained as a particular case. Furthermore, we show that the evolution of demand patterns, while being affected by differential rates of productivity growth in different sectors of the economy, also play an important role in establishing the pace of technical progress.

**JEL Classifications:** O19, F12.

**Keywords:** Cumulative causation, structural change, Kaldor-Verdoorn law.

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## 1. Introduction

While structural change and economic growth register as interrelated processes, the mainstream assigns a key role to issues such as technical progress and capital accumulation, relegating changes in structure to a secondary position in explaining economic growth. The traditional Neoclassical approach, with its emphasis on the supply side, and originally built in terms of one or two sector models [see e.g. Solow (1956), Swan (1956) and Uzawa (1961)] cannot take into account the possible links between growth and changes in the structure of an economy<sup>1</sup>. According to this view, structural change is simply a by-product of the growth in per capita gross domestic product (GDP) [see McCombie (2006) and McMillan and Rodrik (2011)].

This can be sharply contrasted with the post-Keynesian view, where structural change is central to economic development. Different approaches have taken into account the connections between growth and change in this tradition, with particular emphasis on the role played by demand, even in the long run [see e.g. Pasinetti<sup>2</sup> (1981, 1983), Setterfield (2010), Thirlwall (2013) and Ocampo et al. (2009)]. Within this tradition, the Structural Economic Dynamic (SED) view is distinguishable by its simultaneous consideration of supply and demand in a multi-sectoral framework [see Baranzini and Scazzieri (1990)]; in particular, the interaction between the evolving patterns of demand and technical progress is responsible for dynamics of output, prices

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<sup>1</sup> See the introductory chapter of Arena and Porta (2012) for a survey on the state of the art of the literature on structural change after the renewal of interest by the mainstream.

<sup>2</sup> According to these views, structural change registers not just as by-product of growth, as claimed by the mainstream, but rather plays a central role in spurring growth. The migration of the labour force from diminishing returns activities to increasing return activities may be one of the outcomes of fundamental structural changes that allow developing economies to grow so quickly. [see McMillan and Rodrik (2011)].

and structural transformation of economies in different stages of the development process. In this regard, Pasinetti's emphasis upon demand composition offers a significant qualitative improvement vis-a-vis traditional, aggregated models, which fail to adequately consider the composition of consumption demand, and thus conceal changes in structure.

Although the SED approach provides a sophisticated treatment of structural change, some authors such as Gualerzi (2012) and Araujo (2013) have pointed to the necessity of a more inclusive treatment of the demand side in order to provide a full characterization or even endogenisation of technical progress and structural change<sup>3</sup>. In this article we intend to fill that gap by building a bridge between the SED formulation and the Neo-Kaldorian theory<sup>4</sup> of cumulative causation. With the approach carried out here, in which technical change is endogenized, we intend to show that a disaggregated assessment of the static Harrod foreign multiplier allows us to alleviate the somewhat passive notion that demand plays in the SED approach. To accomplish this task, we conceptualize the notion of a demand regime, a well-known concept from the Kaldorian

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<sup>3</sup> Pasinetti (1993, p.69) himself acknowledges the importance of considering a better treatment of the demand side when questioning the origins of technological progress. According to him: "[t]his means that any investigation into technical progress must necessarily imply some hypothesis on the evolution of consumers' preferences as income increases. Not to make such an hypothesis, and to pretend to discuss technical progress without considering the evolution of demand, would make it impossible to evaluate the very relevance of technical progress and would render the investigation itself meaningless."

<sup>4</sup> There have been some developments of the neo-Kaldorian tradition related to models of balance of payments-constrained growth (BPCG). Araujo and Lima (2007) and Araujo (2013), for instance, have derived versions of the balance of payment constrained growth model. Growth performance is explained by considering how the evolution of patterns of consumption can drive the external sector, with consequences for the overall economy.

literature [see e.g. Setterfield and Cornwall (2002)] by using a multi-sectoral version of the Harrod foreign trade multiplier that is based on an extended version of Pasinetti's pure labour model<sup>5</sup>. Here we use the analysis of Trigg and Lee (2005) as a crucial step to establish the links with the Neo-Kaldorian literature. But we have to extend their analysis to an economy with foreign trade, since the Neo-Kaldorian view assigns to exports a key role in autonomous aggregate demand. According to that view, the dynamism of the export sector may give rise to virtuous cycles of economic growth, not only through its effect on aggregate demand but also due to dynamic economies of scale<sup>6</sup> that accrue from an increase in output.

Hence, the first contribution of this paper is the derivation of the multi-sectoral static Harrod multiplier by extending the Pasinettian model. This derivation allows us to derive a proper demand regime for the model. The sectoral productivity regime departs from Araujo (2013), where sectoral Kaldor-Verdoorn's laws were introduced in Pasinetti's model. With this analysis, we are able to introduce the concepts of growth regimes [see Blecker (2010)] in the SED approach, which also allows us to afford a connection between many of the arguments that underpin the importance of the endogenous concept of economic growth.

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<sup>5</sup> Trigg and Lee (2005) have shown that it is possible to derive a simple multiplier relationship from multi-sectoral foundations in the original version of the Pasinetti model, meaning that a scalar multiplier can legitimately be applied to a multi-sectoral economy.

<sup>6</sup> Cornwall and Cornwall (2002, p. 206) highlighted these mechanisms by considering that the contribution of the external sector to productivity growth is twofold: first it allows the larger scale production methods to improve productivity; second it encourages the adoption of the best available productivity-enhancing technologies.

The second contribution of the paper is to show that an open version of the Pasinettian model to foreign trade, advanced by Araujo and Teixeira (2004), may be seen as a particular case of the multi-sectoral version of the Harrod foreign trade multiplier derived here, the former being equal to the latter when the condition of full employment of the labour force is satisfied<sup>7</sup>. As a consequence, it is shown that the multi-sectoral version of the Harrod foreign trade multiplier generates different levels of production and employment, only one of which will be the full employment level that corresponds to the Pasinettian solution.

In order to emphasize this point, we carry out the formulation of a sectoral demand regime both in terms of the Harrod foreign trade multiplier and in terms of the Pasinettian equilibrium sectoral output. The first analysis is developed under the rubric of the Sectoral Demand Regime (SDR) while the latter is referred as the Structural Economic Dynamic Regime (SEDR). Notwithstanding the Neo-Kaldorian emphasis on the role of effective demand in interacting with productivity in a cumulative sense, the derivation of the SEDR also allows us to take into account the role of demand in generating technical change. Moreover, it brings out that the Neo-Kaldorian analysis may also reap benefits from a disaggregated refinement of its basic framework. Even departing from a somewhat narrower view of cumulative causation which emphasizes only the sectoral aspect of dynamic increasing returns of scale – we arrive at a macroeconomic notion, in which technical change in one sector spurs productivity in other sectors through its effect on per capita income growth [see Young (1928)]. Central

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<sup>7</sup> This registers as a well-known result in the SED framework, and one of the main outcomes of the Pasinettian analysis is that in general it is not fulfilled, meaning that unemployment is the most probable outcome of structural change.

to this development is the concept of Engel's law, according to which an evolving pattern of consumption arises when per capita income grows.

This article is structured as follows. In the next section we present the foundations for a theory of demand-growth relationship. Section 3 derives the multi-sectoral multiplier for an open version of the pure labour Pasinettian model. In the fourth section the demand and productivity regimes are modelled in the Pasinettian framework along with the design of a Structural Economic Dynamic Regime (SEDR). Section 5 concludes.

## **2. The basis for a theory of the Consumption and Growth Relationship**

One of the hallmarks of the Post-Keynesian Economics is the role played by aggregate demand not only in the short run output level but also in the pace of long run output growth [see Setterfield (2002)]. The so-called demand-led-growth theory emphasizes the linkages between demand and productivity growth by highlighting 'learning by doing' and economies of scale as important sources of technical change. Such view has its root in Adam Smith's principle that the 'division of labour depends on the extent of the market' and takes economic growth demand induced rather than resource constrained. According to that view the growth process rests on two main tenets:

- (i) The actual rate of growth is demand determined;
- (ii) The division of labour depends on the extent of the market.

From the first item it is possible to identify the operation of a demand regime, DR from now on, which "describes demand formation, and the relationship in which the latter stands to the growth rate of output" [Setterfield and Cornwall (2002)]. In the formal baseline model due to Dixon and Thirlwall (1973), the DR is depicted in terms of three equations. The first one is a dynamic version of the Harrod foreign trade

multiplier, in which the growth rate of exports, namely  $\hat{x}$ , is the key determinant of the growth rate of output,  $\hat{y}$ , according to  $\hat{y} = m\hat{x}$ , where  $m$  is the dynamic foreign trade multiplier. In such formulation, exports are seen as crucial because they register as the only wholly exogenous component and the key driver of aggregate demand [see Setterfield (2010, p. 394)]. The second equation is a usual export function as advanced by Thirlwall (1979) expressed in growth terms, namely  $\hat{x} = \tau(\hat{p}_d - \hat{p}_f - \hat{e}) + \zeta\hat{z}$ , which considers the growth rate of exports as a function of the dynamics of the real exchange rate,  $\hat{p}_d - \hat{p}_f - \hat{e}$ <sup>8</sup>, weighted by the price elasticity of demand,  $\tau$ , and the growth rate of international output,  $\hat{z}$ , weighted by the income elasticity of demand,  $\zeta$ . The third equation within the DR is related to the growth rate of domestic prices as a function of the differential between the growth rate of wages,  $\hat{w}$ , and productivity,  $\hat{q}$ , a relation that accrues from the usual mark-up pricing equation from the Kaleckian growth theory [see Setterfield (2010)]:  $\hat{p}_d = \hat{w} - \hat{q}$ <sup>9</sup>. After some algebraic manipulation, which consists in substituting the last three expressions in to the dynamic version of the foreign Harrod trade multiplier and rearranging terms, is possible to establish the DR as:  $\hat{y} = \Omega + \Gamma\hat{q}$ , where  $\Omega = m\hat{q}_f + \zeta\hat{z}$ <sup>10</sup> and  $\Gamma = -m\tau$ .

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<sup>8</sup>  $\hat{p}_d$  and  $\hat{p}_f$  stand for the growth rate of domestic prices and international prices, respectively, and  $\hat{e}$  stands for the changes in the exchange rate.

<sup>9</sup> It is assumed that the growth rate of foreign prices is determined in the same way as it is in the domestic economy, according to  $\hat{p}_f = \hat{w}_f - \hat{q}_f$ , where  $\hat{w}_f$  is the growth rate of wages abroad and  $\hat{q}_f$  is the productivity growth. In the Kaldorian formulation such variables are assumed exogenous and given.

<sup>10</sup> Such result is obtained under the assumptions that  $\hat{w}_f = \hat{w}$ , which takes into account Kaldor's stylized fact of constant wage relativities, and  $\hat{e} = 0$  [Setterfield (2010)].



The second item refers to a productivity regime, PR hereafter, which describes productivity growth as a function of the growth rate of output according to a Kaldor-Verdoorn function [see Setterfield and Cornwall (2002)]:  $\hat{q} = \alpha + \beta\hat{y}$ . In such formulation productivity growth,  $\hat{q}$ , depends both on an autonomous component,  $\alpha$ , embodying the notion of exogenous productivity determined by the current institutional regime, and on the Verdoorn coefficient,  $\beta$ , which captures the effect of the output growth rate on the productivity growth, conveying the idea of ‘learning by doing’.

From the interaction between the DR and PR, which in practice consists in solving mathematically the above mentioned system with two equations and unknowns, namely  $\hat{y}$  and  $\hat{q}$ , it is then possible to establish both the growth rate of output and productivity. An important outcome of this analysis is that the growth process emerges as a self-reinforcing phenomena in which an exogenous increase in the growth of exports affects positively the growth rate of output, also positively affecting the productivity growth – via the Kaldor-Verdoorn relation – which by its turn increases competitiveness, leading to higher exports and consequently to a better output growth performance. Such process then gives rise to a virtuous cycle of growth, in which the recursive interaction between supply and demand generates a genuinely endogenous growth theory.

In face of the relevance of such developments, by ignoring the conception of endogenous productivity growth associated with the neo-Kaldorian tradition, Pasinetti’s SED approach overlooks the importance of the recursive interaction between supply and demand in a growth scheme. In this vein, if on the one hand the Pasinettian model emphasizes the main channels of interdependence between economic growth and structural change, on the other hand it overlooks the emphasis of demand-led-growth

theory in which consumption and growth feedback in a cumulative process. [Roberts and Setterfield (2007)]. Therefore, by developing such analysis within the Pasinettian multi-sector growth framework allows us to make it sensitive to developments from the demand-led-growth theory. Such endeavor has been pointed out by some authors such as Gualerzi (2012) and Araujo (2013) as an important development to be accomplished within the SED tradition.

In an effort of advancing a theory of consumption-growth relationship, Gualerzi (2012) for instance notes that the SED is an approach rooted in the theory of demand-led-growth insofar as demand matters shape how supply factors and technical change in particular will evolve, not only in the short but also in the long run. But elsewhere the author points out that: “the integration of the demand side into the analysis of growth, which is potentially the most fruitful step forward, does not lead to an analysis of the endogenous growth mechanisms because of a fully inadequate theory of demand” [Gualerzi (1996, p. 157)].

According to that view, the inadequacy of the theory of demand in the SED framework is due to the fact that demand still plays a somewhat passive role since increases in per capita income are motivated by technical change, which is wholly exogenous<sup>11</sup>. Admittedly, in the original version of the Pasinettian model, structural change arises as the by-product of growth and not the other way round: what should be expected from an approach rooted in the Cambridge tradition. Being the focus of Pasinetti’s analysis on the effect of productivity growth differentials over sectoral

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<sup>11</sup> This view is also emphasized by Silva and Teixeira (2008, p.286) : “Although Pasinetti relates both factors with the learning principle, learning itself is essentially unexplained and therefore the question of what moves the driving forces of the economy remains unanswered.”

dynamics, exogenous technical progress hinders a deeper understanding of the endogenous growth mechanisms as emphasized by the demand-led-growth theory.

But this is not the whole story: if on one hand the the Kaldorian literature on cumulative causation is precise in establishing the connections between productivity and output growth, on the other hand it is not able to address another central aspect of the connection between growth and demand as emphasized by Gualerzi (2012). According to the author, the question of how the composition of demand interacts with the growth rate of output in the generation of technical change is as much important as the output and productivity link emphasized by the neo-Kaldorian literature.

The main reason why the neo-Kaldorian literature does not consider this deeper connection between composition of demand and output growth is that its formal model considers national economies in the aggregate. Hence such framework cannot take into account the role of composition of demand on growth by the single reason that the economy is aggregated in one sector. This fact contrasts with the emphasis assigned by Kaldor himself to the role of structure and demand in an open economy, as expressed by McCombie and Thirlwall (1994, p. 164): “Nicholas Kaldor has argued in many of his writings that it is impossible to understand the growth and development process (and divisions between rich and poor countries within the world economy) without taking a sectoral approach, distinguishing between increasing returns activities on the one hand and diminishing returns activities on the other.”

In this sense, by making use of a multi-sectoral approach a proper theory of the consumption-growth relationship can be addressed by highlighting the changing pattern of demand as one of the driving forces of technological improvements, and then considering not only the growth rate of demand as one the determinants of the technological pace. In Gualerzi’s words (2012, p. 21) “change in consumption patterns

should be seen as an essential aspect of the income creation process, rather than an almost automatic effect of income growth”.

One of the examples of the role played by the composition effect on the growth phenomena is offered by the multi-sectoral version of the Thirlwall’s law, another development of the Kaldorian literature on the effects of international trade over the growth performance. Araujo and Lima (2007) have shown that a disaggregated assessment of the balance of payments constrained growth hypothesis advanced by Thirlwall (1979) highlights an aspect of the growth performance that is not evidenced by the aggregated version: not only the elasticity of exported and imported goods along with the growth rate of international income matter for the determination of the output growth performance, but also the share of each good in exports and imports.

This instance shows that there are also benefits from cross-fertilization between the SED and the neo-Kaldorian theories from the view point of the latter approach. This can be grasped by considering that in fact there are another mechanism through which demand may affect technical change and thus economic growth. It is related to the composition effect as emphasized by Gualerzi (2012) and points to the need of considering not only the effects of the growth rate of demand but also of its composition on the growth rate of output. One of the aims of the present paper is to contribute to the development such mechanism by introducing the concept of a DR in the SED approach. By considering simultaneously the existence of a PR in such framework, allows us to determine the sectoral rates of technical change then by filling the gap of an endogenous growth theory for the Pasinettian model.

But we acknowledge that this is just part of the endeavour since a more inclusive treatment of the connections between demand and growth should take into account

another dimension of demand creation that is related to the process of ‘creative destruction’ as advanced by Schumpeter. Such dimension involves a third component in the relation between demand and growth, namely investment. And investment understood not only in terms of the existing goods and methods of production but also in terms of innovation and discovery of new methods of production. Here, our aim is narrower in the sense that we focus only on establishing formally the concepts of DR and PR in the Pasinettian framework where the number of sectors is arbitrarily fixed. In such a scheme, the process of creative destruction cannot be fully taken into account.

### **3. The Derivation of the Multi-sectoral Multiplier for an open economy**

In order to develop a DR in the Pasinettian approach, we depart from Trigg and Lee (2005)<sup>12</sup>, who derived a multi-sectoral version of the Keynesian multiplier. This is a natural step since the DR in the Neo-Kaldorian model is developed in terms of the growth rates of the output given by the dynamic Harrod foreign trade multiplier. But due to the importance of foreign demand in the Neo-Kaldorian literature we go a step further by developing an extended version of the disaggregated multiplier<sup>13</sup> that takes into account international trade. Let us consider an extended version of the Pasinettian model to foreign trade as advanced by Araujo and Teixeira (2004). The starting point is

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<sup>12</sup>The idea of developing a multi-sectoral version of the Keynesian multiplier dates back to Goodwin (1949) and Miyazawa (1960), who accomplished to a disaggregated version of the income multiplier in Leontief’s framework from the relatively simple Keynesian structure. Both authors emphasized that although there are important differences between the Keynes and Leontief approaches, a bridge between them, namely a disaggregated version of the multiplier, is important for the development of both views.

<sup>13</sup> The procedure adopted here is similar to the Pasinettian analysis.

the Pasinetti model of pure labour, where labour is the sole factor of production [see Pasinetti (1993)]. Demand and productivity vary over time at a particular rate in each sector of the two countries – the advanced country is denoted by  $A$  and the underdeveloped country by  $U$ . Assume also that both countries produce  $n - 1$  consumption goods, but with different patterns of production and consumption. From the viewpoint of country  $U$ , and following the notation of Araujo and Teixeira (2004), the system of physical quantities may be expressed as:

$$\begin{cases} X_i - (a_{in} + \xi a_{i\hat{n}})X_n = 0 \\ X_n - \sum_{i=1}^{n-1} a_{ni}X_i = 0 \end{cases} \quad (1)$$

where  $X_i$  denotes the domestic physical quantity produced of consumption good  $i$ ,  $i = 1, \dots, n-1$ , and  $X_n$  represents the quantity of labour in all internal production activities; per capita demand of consumption goods is represented by a set of consumption coefficients: both  $a_{in}$  and  $a_{i\hat{n}}$  stand for the demand coefficients of final commodity  $i$ ,  $i = 1, \dots, n-1$ . The former refers to domestic and the latter to foreign demand. The labour coefficients of consumption goods are represented by  $a_{ni}$ ,  $i = 1, \dots, n-1$ . Furthermore, the family sector in country  $A$  is denoted by  $\hat{n}$  and the population sizes in both countries are related by the coefficient of proportionality  $\xi$ . The first  $n - 1$  equations in the above system refer to the equilibrium in the consumption goods sectors: all production,  $X_i$ , is either consumed internally,  $a_{in}X_n$ , or abroad,  $\xi a_{i\hat{n}}X_n$ ; and the expression in the last line of system (1) represents equilibrium in the labour market. The quantity of labour in all internal production activities,  $X_n$ , is employed in the production of consumption goods. This characterization of the equilibrium does not mean that it will hold throughout the period covered by the

analysis but Pasinetti (1981) assumes that it holds at time zero. The above system may be written in matrix form as:

$$\begin{bmatrix} \mathbf{I} & -(\mathbf{c} + \xi \hat{\mathbf{c}}) \\ -\mathbf{a} & 1 \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ X_n \end{bmatrix} = \begin{bmatrix} \mathbf{O} \\ 0 \end{bmatrix} \quad (2)$$

where  $\mathbf{I}$  is an  $(n-1) \times (n-1)$  identity matrix,  $\mathbf{O}$  is an  $(n-1)$  null vector,  $\mathbf{X} = \begin{bmatrix} X_1 \\ \vdots \\ X_{n-1} \end{bmatrix}$  is the

$(n-1)$  column vector of physical quantities,  $\mathbf{c} = \begin{bmatrix} a_{1n} \\ \vdots \\ a_{n-1,n} \end{bmatrix}$  is the  $(n-1)$  column vector of

consumption coefficients,  $\hat{\mathbf{c}} = \begin{bmatrix} a_{1\hat{n}} \\ \vdots \\ a_{n-1,\hat{n}} \end{bmatrix}$  refers to the  $(n-1)$  column vector of foreign

demand coefficients, and  $\mathbf{a} = [a_{n1} \ \cdots \ a_{n-1,n}]$  is the  $(n-1)$  row vector of labour coefficients. System (2) is a homogenous and linear system and, hence a necessary condition to ensure non-trivial solutions of the system for physical quantities is:

$$\det \begin{bmatrix} \mathbf{I} & -(\mathbf{c} + \xi \hat{\mathbf{c}}) \\ -\mathbf{a} & 1 \end{bmatrix} = 0 \quad (3)$$

Condition (3) may be equivalently written as:

$$\mathbf{a}(\mathbf{c} + \xi \hat{\mathbf{c}}) = 1 \quad (4)$$

By using summations it is possible to rewrite expression (4) as [see Araujo and Teixeira (2004)]:

$$\sum_{i=1}^{n-1} a_{ni} (a_{in} + \xi a_{i\hat{n}}) = 1 \quad (5)$$

If condition (5) is fulfilled then there exists solution for the system of physical quantities in terms of an exogenous variable, namely the full employment population  $\bar{X}_n$ . In this case, the solution of the system for physical quantities may be expressed as:

$$\begin{bmatrix} \mathbf{X} \\ X_n \end{bmatrix} = \begin{bmatrix} (\mathbf{c} + \xi \hat{\mathbf{c}}) \bar{X}_n \\ \bar{X}_n \end{bmatrix} \quad (6)$$

In order to particularize the production in one of the countries let us introduce the superscript  $U$  to denote the components of vector  $\mathbf{X}$  in the underdeveloped country, according to:

$$\begin{cases} X_i^U = (a_{in}^U + \xi a_{in}^U) \bar{X}_n \\ X_n = \bar{X}_n \end{cases} \quad (7)$$

From the first  $n - 1$  lines of (7), we conclude that in equilibrium the physical quantity of each tradable commodity to be produced in country  $U$ , that is  $X_i^U$ ,  $i = 1, \dots, n - 1$ , will be determined by the sum of the internal and foreign demand, namely  $a_{in}^U \bar{X}_n$  and  $\xi a_{in}^U \bar{X}_n$  respectively. The last line of (7) shows that the labour force is fully employed. It is important to emphasize that solution (7) holds only if condition (5) is fulfilled. If (5) does not hold, then the non-trivial solution of physical quantities cannot be given by expression (7). In order to explore the possibility of other meaningful solutions, let us rewrite the system of physical quantities in (2) as:

$$\begin{bmatrix} \mathbf{I} & -\mathbf{c} \\ -\mathbf{a} & 1 \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ X_n \end{bmatrix} = \begin{bmatrix} \mathbf{E} \\ 0 \end{bmatrix} \quad (8)$$



where  $\mathbf{E} = \xi \bar{X}_n \hat{\mathbf{c}}$  denotes the vector of sectoral exports. We may rewrite system (8) as<sup>14</sup>:

$$\begin{cases} \mathbf{X} - \mathbf{c}X_n = \mathbf{E} \\ -\mathbf{a}\mathbf{X} + X_n = 0 \end{cases} \quad (9)$$

From the last line of system (9), it follows that:

$$X_n = \mathbf{a}\mathbf{X} \quad (10)$$

Note that now the employment level is not exogenous as in (7) since we are solving the system by considering the possibility of unemployment. In (7), under a non-trivial solution, a posited exogenous level of full employment was guaranteed. We can now derive a scalar multiplier for the extended Pasinetti system. By pre-multiplying throughout the first line of (9) by  $\mathbf{a}$ , one obtains:

$$\mathbf{a}\mathbf{X} - \mathbf{a}\mathbf{c}X_n = \mathbf{a}\mathbf{E} \quad (11)$$

By substituting (10) into expression (11), and isolating  $X_n$ , one obtains the employment multiplier relationship:

$$X_n = \frac{1}{1 - \mathbf{a}\mathbf{c}} \mathbf{a}\mathbf{E} \quad (12)$$

where  $1/1 - \mathbf{a}\mathbf{c}$  is a scalar employment multiplier [Trigg and Lee (2005)]. Using the expression  $\mathbf{E} = \xi \bar{X}_n \hat{\mathbf{c}}$ , (12) can be written using summation notation as:

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<sup>14</sup> Dealing with the original Pasinettian model, Trigg and Lee (2005) had to assume that investment in the current period becomes new capital inputs in the next period and that the rate of depreciation is 100% (that is, all capital is circulating capital) in order to derive the Keynesian multiplier. See Harcourt (1965) for a similar take on investment. By considering an economy extended to foreign trade we do not need this hypothesis.

$$X_n = \frac{\sum_{i=1}^{n-1} \xi a_{ni}^U a_{in}^U}{1 - \sum_{i=1}^{n-1} a_{ni}^U a_{in}^U} \bar{X}_n \quad (12)'$$

This result shows that if the effective demand condition given by expression (5) is fulfilled then the employment level is equal to the full employment level. This is the content of:

**Proposition 1:**

If condition (5) holds then the employment level generated by expression (12)' is equal to the full employment level, namely  $X_n = \bar{X}_n$ .<sup>15</sup>

As is shown in Proposition 1, while expression (12)' generates different levels of employment, only one of them will be the full employment level that corresponds to the Pasinettian solution. Through further decomposition [see Trigg (2006, Appendix 2)], (12) can be substituted into the first line of (9) to yield:

$$\mathbf{X} = \left( \mathbf{I} + \frac{\mathbf{ca}}{1 - \mathbf{ac}} \right) \mathbf{E} \quad (13)$$

This is a multiplier relationship between the vector of gross outputs,  $\mathbf{X}$ , and the vector representing final demand  $\mathbf{E}$ , where  $\left( \mathbf{I} + \frac{\mathbf{ca}}{1 - \mathbf{ac}} \right)$  is the output multiplier matrix. One of the main differences between this multi-sectoral multiplier for an open economy and the one derived by Trigg and Lee is that the latter is a scalar, and the former is a matrix.

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<sup>15</sup> See the Appendix for the proofs of propositions 1 to 3.

That result is akin to a multi-sectoral version of the Harrod foreign trade multiplier<sup>16</sup> whereby the output of each sector is related to the export performance of that sector. Such expression shows that the composition of the structure of production also matters for income determination. A country with access to foreign markets may induce changes in the structure of production that will allow the reallocation of resources from the low to high productivity sectors, thus giving rise to a propitious economic structure that will lead to higher output. After some algebraic manipulation [see Appendix] it is possible to rewrite expression (13) as:

$$X_i^U = \left( a_{in}^U \frac{\sum_{i=1}^{n-1} \xi a_{ni}^U a_{in}^U}{1 - \sum_{i=1}^{n-1} a_{ni}^U a_{in}^U} + \xi a_{in}^U \right) \bar{X}_n \quad (14)$$

Expression (14) plays a central role in our analysis. It shows that the solution given by multi-sectoral Harrod foreign trade multiplier for output of the  $i$ -th sector is due to two components: the domestic demand, conveyed by the domestic consumption coefficient  $a_{in}^U$ , and external demand, portrayed by the foreign demand coefficient  $a_{in}^U$ . Due to reasons that will become clearer latter, the domestic coefficient is affected by the structural economic dynamics of the economy as a whole, captured by the quotient:

$$\frac{\sum_{i=1}^{n-1} \xi a_{ni}^U a_{in}^U}{1 - \sum_{i=1}^{n-1} a_{ni}^U a_{in}^U}. \text{ This quotient differentiates the solution obtained here, given by}$$

expression (14), from the solution (7) derived by Araujo and Teixeira (2004) for an

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<sup>16</sup> The static Harrod foreign trade multiplier [Harrod (1933)] is given by:  $Y = \frac{1}{m} X$ , where  $Y$  denotes

income,  $X$  exports and  $m$  is the marginal propensity to import. According to this expression the main determinant of income is the level of export demand in relation to the propensity to import.

open-economy version of the Pasinettian model. We can prove that solution (7) is a particular case of solution (14) when condition (5) holds. In other words:

**Proposition 2:**

The equilibrium solution given by the  $n - 1$  first lines of expression (7) is a special case of the solution given by multi-sectoral Harrod foreign trade multiplier (14) when the full employment condition (5) is satisfied.

Proposition 2 shows that the solution put forward by Araujo and Teixeira (2004) for an open version of the Pasinetti model is in fact a special case of the solution obtained here. That result is of key importance. One of the central results of the SED analysis [See Pasinetti (1981, 1993)] is that even departing from an equilibrium position, where full employment prevails, condition (5) will not hold in the long run due to the particular dynamics of technical progress and evolution of demand for each sector. It means that, in general, one should expect that:  $\sum_{i=1}^{n-1} a_{ni}^U (a_{in}^U + \xi a_{in}^U) < 1$ . We may

consider a symmetrical case, namely  $\sum_{i=1}^{n-1} a_{ni}^U (a_{in}^U + \xi a_{in}^U) > 1$ , which corresponds to the case of overemployment. Then we have the following proposition:

**Proposition 3:**

If  $\sum_{i=1}^{n-1} a_{ni}^U (a_{in}^U + \xi a_{in}^U) < 1$  then the production given by disaggregated Harrod foreign multiplier (14) is lower than equilibrium Pasinettian production given by expression (7). Otherwise, (14) is higher than (7).

In sum, the amount of structural unemployment cannot be defined independently of the level of aggregate demand and one should expect that the sectoral output given by

the Harrod foreign trade multiplier can deviate from the equilibrium output. In the Pasinettian analysis the case in which  $\sum_{i=1}^{n-1} a_{ni}^U (a_{in}^U + \xi a_{in}^U) < 1$  receives more attention, since one of the probable outcomes of structural change is structural unemployment. From these Propositions, it is possible to conclude that a country may experience balance of payment equilibrium with levels of employment and production lower than those related to full employment and equilibrium. Hence a poor export performance may lead to low levels of employment and sectoral output thus showing that the external constraint may be more relevant than shortages in savings and investment mainly for developing economies. In this context the disaggregated version of the Harrod foreign trade multiplier plays a decisive role since it changes the focus of determination of national income from investment to exports.

The inclusion of a government sector may give rise to another triggering source of the virtuous cycle of growth if the government consumption is taken as an exogenous variable. Such variable can play the same role of the exogenous exports. In the neo-Kaldorian literature, however, exports are assigned the central role because it is the wholly autonomous component of aggregate demand. Other components such as investment and Government expenditures are to some extent endogenous to the income determination, but their autonomous part may also be adopted to trigger the cumulative mechanism. In the present paper we do not consider this other sources since a disaggregated treatment of them should be provided what is beyond the scope of the paper. In the analysis that follows we will make use of the results (7) and (14) to derive growth regimes in the SED approach.

## 4. Macroeconomic Regimes in a Structural Economic Dynamic Approach

### 4.1. The Sectoral Demand Regime (SDR)

In order to determine the sectoral growth rate of output from expression (14) one has to specify the dynamic path of terms of trade since price competitiveness plays a crucial role in the theory of cumulative causation. Following Araujo (2013) we consider that  $p_i^U$  and  $p_i^A$  stand for prices of the  $i$ -th consumption good in countries  $U$  and  $A$ , respectively. By considering that  $e$  stands for the nominal exchange rate in the  $U$  country, we also consider that per capita export coefficient  $a_{in}^U$  is given according to one of the following possibilities:

i) On one hand, if  $ep_i^A < p_i^U$ , that is, if country  $U$  has no comparative cost advantage in the production of consumption good  $i$ , then the per capita foreign demand for good  $i$  is assumed to be zero:  $a_{in}^U = 0$ . If  $ep_i^A \geq p_i^U$ , then let us consider that the foreign demand for the consumption good  $i$  is given by an export function *à la* Thirlwall (1979) [see Araujo and Lima (2007)]:

$$a_{in}^U = \left( \frac{p_i^U}{ep_i^A} \right)^{\eta_i} y_A^{\beta_i} X_n^{1-\beta_i} \quad (15)$$

where  $y_A$  denotes the per capita income of country  $A$ . While  $\eta_i$  designates a price elasticity of demand for exports of good  $i$ , with  $\eta_i < 0$ ,  $\beta_i$  denotes an income elasticity of demand for exports, with  $\beta_i > 0$ . According to this specification, it is not assumed *ex-ante* full specialization.

ii) On the other hand, if country  $A$  has no comparative cost advantage in the production of consumption good  $i$ , we assume country  $U$  does not import that good, that is,  $a_{in}^U = 0$

, where  $a_{in}$  stands for the per capita import coefficient for good  $i$ . But if  $p_i^U \geq ep_i^A$ , we consider that the demand coefficients for imports are given by the following import function:

$$a_{in}^U = \left( \frac{ep_i^A}{p_i^U} \right)^{\phi_i} y_U^{\phi_i} X_n^{\phi_i - 1} \quad (16)$$

where  $\phi_i$  is the price elasticity of the demand for imports of good  $i$ , with  $\phi_i < 0$ ,  $\phi_i$  is the income elasticity of the demand for imports of good  $i$  and  $y_U$  is the per capita income of country  $U$ . Following Pasinetti (1981), the coefficients of internal demand  $a_{in}^U$ 's are assumed to vary according to:

$$a_{in}^U(t) = a_{in}^U(0)e^{r_i^U t} \quad (17)$$

where  $r_i^U$  stands for the growth rate of domestic demand of good  $i$  in the  $U$  country. In what follows we assume that the evolution of consumption patterns is endogenous considering that the growth rate of sectoral demand is a function, not only of technical coefficients,  $a_{ni}^U$ , but also of their variations [see Pasinetti (1981, 1993)]. From expressions (15) and (17) and by adopting the following convention:  $\frac{\dot{p}_i^U}{p_i^U} = \sigma_i^U$ ,

$\frac{\dot{p}_i^A}{p_i^A} = \sigma_i^U$ ,  $\frac{\dot{e}}{e} = \varepsilon$ ,  $\frac{\dot{y}_A}{y_A} = \sigma_y^A$ , and  $\frac{\dot{X}_{\hat{n}}}{X_{\hat{n}}} = \hat{g}$  we conclude that the growth rate of foreign and home demand for consumption good  $i$  are given respectively by:

$$\frac{\dot{a}_{in}^U}{a_{in}^U} = \eta_i(\sigma_i^U - \sigma_i^A - \varepsilon) + \beta_i \sigma_y^A + (1 - \beta_i)g \quad (18)$$

$$\frac{\dot{a}_{in}^U}{a_{in}^U} = r_i^U \quad (19)$$

In what follows, we consider that the growth rate of foreign demand for the  $i$ -th consumption good is denoted by  $\hat{r}_i = \eta_i(\sigma_i^U - \sigma_i^A - \varepsilon) + \beta_i\sigma_y^A + (1 - \beta_i)g$ . Following Araujo and Teixeira (2004) domestic and foreign prices are given by:

$$p_i^U(t) = a_{ni}^U(t)w^U \quad (20)$$

$$p_i^A(t) = a_{\hat{n}\hat{i}}^A(t)w^A \quad (21)$$

where  $w^U$  and  $w^A$  stand for the wages in countries  $U$  and  $A$ , respectively, and  $a_{ni}^A(t)$  stands for the labour coefficient of the  $i$ -th sector in country  $A$ . According to that formulation, prices are given by the costs of production. The dynamics of technical coefficients, namely  $a_{ni}^U$  and  $a_{\hat{n}\hat{i}}^A$ , in countries  $U$  and  $A$  are given respectively as:

$$a_{ni}^U(t) = a_{ni}^U(0)e^{-\rho_i^U t} \quad (22)$$

$$a_{\hat{n}\hat{i}}^A(t) = a_{\hat{n}\hat{i}}^A(0)e^{-\rho_i^A t} \quad (23)$$

where  $\rho_i^U$  is the rate of technical progress in  $i$ -th sector of  $U$  country and  $\rho_i^A$  represents technical progress in  $i$ -th sector of country  $A$ . Hence, from (22) and (23):

$$\frac{\dot{a}_{ni}^U}{a_{ni}^U} = -\rho_i^U \quad (24)$$

$$\frac{\dot{a}_{\hat{n}\hat{i}}^A}{a_{\hat{n}\hat{i}}^A} = -\rho_i^A \quad (25)$$

By taking logs, and differentiating expressions (22) and (23) in relation to time, one obtains the dynamics of prices as given by:

$$\sigma_i^U = \sigma_w^U - \rho_i^U \quad (26)$$



$$\sigma_i^A = \sigma_w^A - \rho_i^A \quad (27)$$

where  $\sigma_w^U$  and  $\sigma_w^A$  stand for the growth rates of wages in countries  $U$  and  $A$  respectively. By taking logs and differentiating expression (14) and considering expressions (24), (25), (26), and (27) one obtains the growth rate of the production of the  $i$ -th sector as:

$$\begin{aligned} \frac{\dot{X}_i^U}{X_i^U} = & \Pi_i^U \left\{ \left( \hat{r}_i^U + g \right) + \xi a_{in}^U X_n \left( 1 - \sum_{j=1}^{n-1} a_{nj}^U a_{jn}^U \right) \left[ \sum_{j=1}^{n-1} \left( \hat{r}_j^U + r_i^U + g - \rho_j^U \right) a_{nj}^U a_{jn}^U a_{in}^U X_n \right] \right\} + \\ & + \left( 1 - \Pi_i^U \right) \frac{\sum_{j=1}^{n-1} \left( r_j - \rho_j^U \right) a_{jn}^U a_{nj}^U}{1 - \sum_{j=1}^{n-1} a_{jn}^U a_{nj}^U} \end{aligned} \quad (28)$$

$$\text{Where } \Pi_i^U = \frac{\xi a_{in}^U X_n}{\xi a_{in}^U X_n + \left( 1 - \sum_{i=1}^{n-1} a_{in}^U a_{ni}^U \right)^{-1} \left( \sum_{j=1}^{n-1} a_{nj}^U a_{jn}^U a_{in}^U X_n \right)}.$$

In order to make a parallel with the Neo-Kaldorian literature, in what follows let us rewrite expression (28), namely  $\frac{\dot{X}_i^U}{X_i^U}$ , as a linear function of technical progress of the

$i$ -th sector:

$$\frac{\dot{X}_i^U}{X_i^U} = \Gamma_i^{SDR} \rho_i^U + \Omega_i^{SDR} \quad (29)$$

$$\text{Where: } \Gamma_i^{SDR} = -\Pi_i \eta_i + \frac{(1 - \Pi_i) a_{ni}^U a_{in}^U}{\sum_{j=1}^{n-1} a_{nj}^U a_{jn}^U} - \frac{(1 + \eta_i)}{\left( 1 - \sum_{j=1}^{n-1} a_{nj}^U a_{jn}^U \right) \xi a_{in}^U X_n + \sum_{j=1}^{n-1} a_{nj}^U a_{jn}^U} \text{ and}$$

$$\begin{aligned}
\Omega_i^{SDR} = & \Pi_i [\eta_i (\rho_i^A - \varepsilon) + \beta_i \sigma_y^A + g] + \\
& + \frac{1}{\left(1 - \sum_{j=1}^{n-1} a_{jn}^U a_{nj}^U\right)} \left\{ \frac{(1 - \Pi_i) \left[ \sum_{\substack{j=1 \\ j \neq i}}^{n-1} (r_j - \rho_j^U) a_{jn}^U a_{nj}^U - r_i^U a_{ni}^U a_{in}^U \right] + \eta_i (\rho_i^A - \varepsilon) + \sum_{\substack{j=1 \\ j \neq i}}^{n-1} \eta_j (\rho_j^A - \rho_j^U - \varepsilon) + \beta_i \sigma_y^A + g}{\left(1 - \sum_{j=1}^{n-1} a_{jn}^U a_{nj}^U\right) \left( \sum_{j=1}^{n-1} a_{nj}^U a_{jn}^U a_{in}^U X_n \right) + \xi a_{in} X_n} \right\}
\end{aligned}
\tag{30}$$

Expression (29) is the sectoral counterpart of the DR, derived from a multi-sectoral Harrod multiplier. We label this solution as the Sectoral Demand Regime (SDR), and it expresses the growth rate of the  $i$ -th sector as a function of technical progress. In order to fully determine the pace of technical progress and the growth rate of demand for the  $i$ -th sector, one also has to advance the notion of a productivity regime in a multi-sectoral set-up. This task is accomplished in the next subsection.

#### 4.2. The Sectoral Productivity Regime (SPR)

In order to establish the sectoral counterpart of the PR, namely a sectoral productivity regime – SPR hereafter – let us assume following Araujo (2013) that the sectoral growth rate of productivity is given by sectoral Kaldor-Verdoorn laws. According to that view, the dynamic economies of scale result from the increasing specialization of labor provided by sectoral market growth, and from the productivity gains that accrues from the learning by doing. Hence:

$$\rho_i^U = \frac{\dot{q}_i^U}{q_i^U} = \gamma_i^U + \alpha_i^U \frac{\dot{X}_i^U}{X_i^U}
\tag{31}$$

Where  $\rho_i^U$  is the rate of technical progress in  $i$ -th sector of  $U$  country,  $\gamma_i^U$  is the intercept of the Verdoorn relation, and  $\alpha_i^U$  poses itself as the Verdoorn coefficient. It does not matter if the production increases occur at the firm level – that is, if they are

restricted to one of the firms in a sector – or if they are widespread amongst firms. Both the individual firm and the aggregated sectoral production play an important role in the generation of sectoral productivity gains. Expression (31) may be rewritten as:

$$\frac{\dot{X}_i^U}{X_i^U} = -\frac{\gamma_i^U}{\alpha_i^U} + \frac{1}{\alpha_i^U} \rho_i^U \quad (31)'$$

Expression (31)' plays the role of a PR in our formulation. By equalizing expression (31)' to (29), namely the SDR to SPR, it is possible to obtain after some algebraic manipulation the rate of technical progress in the  $i$ -th sector as:

$$(\rho_i^U)_{SDR}^* = \frac{\gamma_i^U + \alpha_i^U \Omega_{SDR}}{1 - \alpha_i^U \Gamma_{SDR}} \quad (32)$$

Expression (32) conveys one of the important outcomes of this analysis: technical progress in the  $i$ -th sector, that is  $\rho_i^U$ , is affected by technical progress in other sectors, namely  $\rho_j^U$  which appear in expression (30) given by  $\Omega_i^{SDR}$ . This highlights an important property of the model: when we adopt a more inclusive treatment of demand in Pasinetti's model, the role of productivity spillovers is emphasized according to the Neo-Kaldorian literature. The straight effect of an increase in  $\rho_j^U$  is to increase  $\rho_i^U$ , meaning that positive effects of technical progress in the  $j$ -th sector will not be restricted to that sector, but will affect the generation of technical progress in other sectors. The rationale behind this interaction may be grasped by considering that technical progress in the  $j$ -th sector has a negative effect on the price of good  $j$ . A smaller price for good  $j$  is translated in terms of higher purchasing power, which may be unevenly spent on consumption of other goods, let us say  $i$ . A higher level of consumption for good  $i$  means, through the Kaldor-Verdoorn relation, a higher level of technical progress for

the  $i$ -th sector<sup>17</sup>. By substituting expression (32) into expression (31), one obtains the growth rate of production of the  $i$ -th sector in the  $U$  country as:

$$\left( \frac{\dot{X}_i^U}{X_i^U} \right)_{SDR}^* = \frac{\Gamma_{SDR} \mathcal{V}_i^U + \Omega_{SDR}}{1 - \alpha_i^U \Gamma_{SDR}} \quad (33)$$

The analysis here is similar to the aggregated model. Since we are focusing on a sectoral aspect of the dynamics, let us consider as a device the case in which<sup>18</sup>:  $r_j = \rho_j^U = 0, \forall j \neq i$ . By following this approach we obtain  $\Omega_i^{SDR}$  and  $\Gamma_i^{SDR}$  as constants and a graphical approach may be adopted. In this case,  $\Omega_i^{SDR}$  may be rewritten as:

$$\Omega_i^{SDR} = \Pi_i [\eta_i (\rho_i^A - \varepsilon) + \beta_i \sigma_y^A + g] + \frac{1}{\left( 1 - \sum_{i=1}^{n-1} a_{in}^U a_{ni}^U \right)} \left\{ \frac{(1 - \Pi_i) [-r_i^U a_{ni}^U a_{in}^U] + \eta_i (\rho_i^A - \varepsilon) + \sum_{j=1}^{n-1} \eta_j (\rho_j^A - \varepsilon) + \beta_i \sigma_y^A + g}{\left( 1 - \sum_{i=1}^{n-1} a_{in}^U a_{ni}^U \right) \left( \sum_{j=1}^{n-1} a_{nj}^U a_{jn}^U a_{in}^U X_n \right) + \xi a_{in} X_n} \right\} \quad (30),$$

Hence we plot the SDR and SPR in a graph as follows:

[Figure 1 goes here]

The interpretation of this graph is similar to the traditional Neo-Kaldorian models. If we start with values of  $\rho_i^U$  and  $\frac{\dot{X}_i^U}{X_i^U}$  below their equilibrium values, then the  $i$ -th sector experience a rate of output growth that will induce the pace of technical

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<sup>17</sup> Note that this property was not obtained in the SED version of the endogenised technical progress derived by Araujo (2013).

<sup>18</sup> Although this case is unrealistic it may bring out the properties of our model. Note that Pasinetti (1993) considers in his structural economic dynamics as the first approximation the case in which  $r_i = \rho_i^U, \forall i$ .

progress, leading to higher price competitiveness that by its turn increase the exports. This will lead to a higher rate of sectoral output growth that will induce more productivity gains and further gains in terms of price competitiveness and export performance.

According to this view, structural changes are triggered by exogenous demand that induces technological progress through increasing returns of scale and learning-by-doing. The consequent increase in per capita income due to the raise in productivity will turn into an increase into per capita demand that may also generate higher levels of productivity. In some moment of this virtuous cycle, structural changes are made endogenous. These result underscore the need for a better treatment of demand side in the SED approach as pointed out by Gualerzi (2001, p. 26): “[i]n Pasinetti’s scheme, since the very source of income growth, technical change, is itself fully exogenous, potential demand is identified only with available disposable income; as such it is a passive notion”.

#### **4.3. A Structural Economic Dynamic Regime (SEDR)**

This study was initially developed in order to endogenize technical change in the Pasinettian model and thereby expression (33) is an attempt to fulfil this aim. But further inquiry shows that indeed it generates a simultaneous system of  $n - 1$  variables and equations. If on one hand, this system is useful to evince the connections amongst technical progress in different sectors as advocated by the Neo-Kaldorian literature, on the other hand, the task of determining the pace of technical change for a specific sector becomes cumbersome.

Now we can take advantage of the Pasinettian solution given by expression (7). This possibility was raised by Araujo (2013), under which it is possible, from

expression (7), to derive the growth rate of potential sectoral output in the long run – what we call here as our SEDR in contrast to the SDR. By taking logs and differentiating expression (7) one obtains:

$$\frac{\dot{X}_i^U}{X_i^U} = \theta_i^U \frac{\dot{a}_{in}^U}{a_{in}^U} + (1 - \theta_i^U) \frac{\dot{a}_{in}^U}{a_{in}^U} + \frac{\dot{X}_n}{X_n} \quad (34)$$

where  $\theta_i^U = \frac{a_{in}^U}{a_{in}^U + \xi a_{in}^U}$  stands for the share of internal demand in total demand of good  $i$ ,  $0 \leq \theta_i^U \leq 1$ . By inserting (20) and (21) into expression (34), we obtain after some algebraic manipulation the growth rate of potential output for the  $i$ -th sector as:

$$\frac{\dot{X}_i^U}{X_i^U} = \theta_i^U r_i^U + (1 - \theta_i^U) [\eta_i (\sigma_i^U - \sigma_i^A - \varepsilon) + \beta_i \sigma_y^A + (\beta_i - 1)g] + g \quad (34)'$$

By adopting the same procedure of the previous section, from expression (34)', we can write the growth rate of output in the  $i$ -th sector as a function of technical progress in that sector. Hence expression (34)' may be rewritten as:

$$\frac{\dot{X}_i^U}{X_i^U} = \Gamma_i^{SEDR} \rho_i^U + \Omega_i^{SEDR} \quad (35)$$

where:  $\Gamma_i^{SEDR} = -(1 - \theta_i^U) \eta_i$  and  $\Omega_i^{SEDR} = \theta_i^U r_i^U + (1 - \theta_i^U) [\eta_i (\gamma_i^A + \alpha_i^A \lambda_i^A \sigma_y^A - \varepsilon) + \beta_i \sigma_y^A]$ .

By replacing expression (31), which represents the SPR, into expression (35), we obtain after some algebraic manipulation the growth rate of productivity in the  $i$ -th sector:

$$(\rho_i^U)_{SEDR}^* = \frac{\gamma_i^U + \alpha_i^U \Omega_{SEDR}}{1 - \alpha_i^U \Gamma_{SEDR}} \quad (36)$$

Expression (36) yields the pace of technical progress by considering the interaction between SPR with SEDR. By substituting expression (36) in expression (35)

one obtains after some algebraic manipulation, the equilibrium growth rate of output under the SEDR:

$$\left( \frac{\dot{X}_i^U}{X_i^U} \right)_{SEDR}^* = \frac{\Gamma_{SEDR} \gamma_i^U + \Omega_{SEDR}}{1 - \alpha_i^U \Gamma_{SEDR}} \quad (37)$$

It is worth recalling that while the derivation of the SDR is based on the solution given by multi-sectoral Harrod foreign trade multiplier production [See Dixon and Thirlwall (1975)], the derivation of SEDR is based on potential or equilibrium production, one should expect at least deviation of the production under SDR from the production under SEDR in the short run – see Proposition 3. But in the long-run, one should expect that the growth rate of production given by expressions (33) and (37)

should be equal, that is  $\left( \frac{\dot{X}_i^U}{X_i^U} \right)_{SDR}^* = \left( \frac{\dot{X}_i^U}{X_i^U} \right)_{SEDR}^*$ . The graph below illustrates this point.

[Figure 2 goes here]

Although the intercepts and slopes of the SPR and SEDR are different, there is a point in which they coincide and this corresponds to the long run solution. Following this rationale, the growth rate of output under SEDR and SDR should be equal in the long-run. With the growth rate of output being given in terms of the parameters of the model, it is also possible to establish the pace of technical change in terms of these parameters as in expression (36). In this vein, technical change is endogenized in the Pasinettian model.

When demand in a particular sector is fostered, the productivity in that sector is spurred on due to the Kaldor-Verdoorn effect. But higher productivity is translated into higher real wages, which may give rise to further increases in demand, but not necessarily in demand for the good that kick started the process. Sectors producing

goods with higher income elasticity of demand tend to increase their share in national income insofar as per capita income grows. Hence, those sectors will also enjoy higher rates of technical progress following the cumulative rationale. Finally, the present approach stresses that the triggering point of this virtuous cycle is external demand, but once it is under way, internal demand may expand and may also be an important component to spur growth. In this vein a vigorous strategy of export led growth may play an important role to trigger the virtuous cycle initiated by cumulative causation.

## **5. Concluding Remarks**

Notwithstanding Pasinetti's emphasis on the evolving patterns of demand within a multi-sectoral framework, demand still plays a somewhat passive role in his approach to the extent that its evolution registers as a function of technical progress, which is wholly exogenous. In this vein, although the original SED approach provides a simultaneous approach of demand and supply sides of economic growth, it does not take into account the role played by cumulative causation in the generation of technical progress. The present analysis aims to join these lines of research on structural factors in a more fully specified multi-sectoral framework, in which demand interacts with technical progress.

With this inquiry we have introduced concepts such as demand and productivity regimes in a version of the Pasinettian model extended to formally consider international trade, by showing that indeed it can be treated as a particular case of the multi-sectoral version of the Harrod foreign trade multiplier. That was proven to be a required step to formulate a proper notion of demand regime in the SED framework. Besides, by considering the interaction between demand and productivity regimes, it



was possible not only to endogenise technical progress in the Pasinettian approach but also to highlight the spillover connections between technical change in different sectors.

If on the one hand, endogenous technical progress is required to properly explain the evolving patterns of demand, on the other hand, the evolution of demand is seen as a function of the technical conditions. In this respect, a Neo-Kaldorian approach to the SED is convenient since it allows us to evince the connections between demand and technical change through the use of the cumulative causation concept.

If on the SED front, the gains from considering Neo-Kaldorian concepts are pervasive, also in the Neo-Kaldorian view we may reap some benefits from the cross-fertilization between these two strands. They accrue mostly from the use of a disaggregated model embedded with sectoral Kaldor-Verdoorn's law, thus emphasising the connections between demand and productivity growth not only at an aggregated but also at a disaggregated level. Once there is an exogenous increase of demand in a particular sector, the productivity increases give rise to per capita income gains that are translated into higher demand. This higher per capita income may be translated into higher demand for goods with higher income elasticity of demand, thus generating a virtuous cycle.

One strength of the approach presented here is its emphasis on the role played by demand in the process of economic growth. According to this view, demand cannot be limited to drive structural changes, but it should also be considered as one of the engines of economic growth via its effect on stimulating the creation and diffusion of technical progress.

## References

- Araujo, R. 2013. Cumulative Causation in a Structural Economic Dynamic Approach to Economic Growth and Uneven Development. *Structural Change and Economic Dynamics* 24, 130 – 140.
- Araujo, R. and Lima, G. 2007. A Structural Economic Dynamic Approach to Balance of Payment Constrained Growth. *Cambridge Journal of Economics* 31(5), 755 – 774.
- Araujo, R. and Teixeira, J. 2004. A Pasinettian Approach to International Economic Relations: The Pure Labour Case. *Review of Political Economy*, 16(1), 117 – 129.
- Arena, R. and Porta, P. 2012. Introduction: Structural Dynamics and Contemporary Growth Theory. In: *Structural Dynamics and Economic Growth*. Edited by: R. Arena and P. Porta. Cambridge University Press, New York.
- Baranzini, M. and Scazzieri, R. 1990. *The Economic Theory of Structure and Change*. Cambridge University Press. Cambridge.
- Blecker, R. 2010. Long-run growth in open economies: export-led cumulative causation or a balance-of-payments constraint? Paper prepared for the 2nd Summer School on “Keynesian Macroeconomics and European Economic Policies”, Berlin, Germany, 2-9 August 2009.
- Cornwall, J. and Cornwall, W. 2002. A Demand and Supply Analysis of Productivity Growth. *Structural Change and Economic Dynamics*, 13(2), 203–229.
- Dixon, R. and Thirlwall, A. 1975. A model of regional growth rate differences along Kaldorian lines, *Oxford Economic Papers*, 27, 201 – 214.
- Gualerzi, D. 1996. Natural Dynamics, Endogenous Structural Change and the Theory of Demand: A Comment on Pasinetti. *Structural Change and Economic Dynamic* 7(2), 147 – 162.

- Gualerzi, D. 2001. *Consumption and Growth: Recovery and Structural Change in the US Economy*. Edward Elgar, Cheltenham, UK.
- Gualerzi, D. 2012. Towards a Theory of the Consumption-Growth Relationship. *Review of Political Economy* 24(1), 15 – 32.
- Goodwin, R. 1949. The Multiplier as Matrix. *The Economic Journal*, 59(236), 537 – 555.
- Harcourt, G. 1965. A Two-Sector Model of The Distribution of Income and the Level of Employment in the Short Run. *The Economic Record*, 41(93), 103 – 117.
- Harrod, R. 1933. *International Economics*. Cambridge University Press, Cambridge.
- McCombie, J. 2006. The Nature of Economic Growth and the Neoclassical Approach: More Questions than Answers? In: *Growth and Economic Development: Essays in Honour of A.P. Thirlwall*. Editors: P. Arestis, J. McCombie and R. Vickerman. Edward Elgar, Cheltenham, UK.
- McCombie, J. and Thirlwall, A. 1994. *Economic Growth and Balance-of-Payments Constraint*. St. Martins Press. New York.
- McMillan, M. and Rodrik, D. 2011. Globalization, Structural Change and Productivity Growth. *NBER Working Paper* No. 17143.
- Miyazawa, K. 1960. Foreign Trade Multiplier, Input-Output Analysis and the Consumption Function. *The Quarterly Journal of Economics*, 74(1), 53 – 64.
- Ocampo, J., Rada, C. and Taylor, L. 2009. *Growth and Policy in Developing Countries: A Structuralist Approach*. Columbia University Press.

- Pasinetti, L. 1981. *Structural Change and Economic Growth – A Theoretical Essay on the Dynamics of the Wealth of the Nations*. Cambridge University Press. Cambridge, UK.
- Pasinetti, L. 1993. *Structural Economic Dynamics – A Theory of the Economic Consequences of Human Learning*. Cambridge University Press. Cambridge, UK.
- Roberts, M. and Setterfield, M. 2007. What is Endogenous Growth Theory? In P. Arestis, M. Baddeley and J. McCombie (eds.). *Economic Growth: New Directions in Theory and Policy*, Cheltenham, Edward Elgar, 14 – 31.
- Setterfield, M. 2010. Endogenous Growth: A Kaldorian Approach. Trinity College Department of Economics. Working Paper 10-01.
- Setterfield, M. and Cornwall, J. 2002. A neo-Kaldorian Perspective on the Rise and Decline of the Golden Age. In M. Setterfield, ed., *The Economics of Demand-led Growth: Challenging the Supply-side Vision of the Long Run*. Cheltenham, UK: Edward Elgar.
- Silva, E. and Teixeira, A. 2008. Surveying Structural Change: Seminal Contributions and A Bibliometric Account. *Structural Change and Economic Dynamics* 19, 273–300.
- Solow, R. 1956. A Contribution to the Theory of Economic Growth. *Quarterly Journal of Economics* 70, 65–94.
- Swan, T. 1956. Economic Growth and Capital Accumulation. *Economic Record*, 32 (2), 334–361.
- Thirlwall, A. 1979. The Balance of Payments Constraint as An Explanation of International Growth Rates Differences, *Banca Nazionale del Lavoro Quarterly Review* 128, 45 – 53.

- Thirlwall, A. 1983. A Plain Man's Guide to Kaldor's Growth Law. *Journal of Post Keynesian Economics* V(3), 345 – 358.
- Thirlwall, A. 2013. *Economic Growth in an Open Developing Economy: The Role of Structure and Demand*. Edward Elgar, Cheltenham, UK and Northampton, MA, USA.
- Trigg, A. 2006. *Marxian Reproduction Schema*. London: Routledge.
- Trigg, A. and Lee, F. 2005. Pasinetti, Keynes and the Multiplier. *Review of Political Economy*, 17(1), 29 – 43.
- Uzawa, H. 1961. On a Two-Sector Model of Economic Growth. *Review of Economic Studies*, 29(1), 40 – 47.
- Young, A. 1928. Increasing Returns and Economic Progress. *Economic Journal*, 38 (52), 527 – 542.

## Appendix

Here we show how to depart from the matrix form of the multi-sectoral static Harrod foreign trade multiplier, as stated in expression (13), to obtain the production of the  $i$ -th sector according to this formulation. From expression (13) we know that:

$$\begin{bmatrix} X_1 \\ \vdots \\ X_{n-1} \end{bmatrix} = \begin{bmatrix} a_{1n} \\ \vdots \\ a_{n-1,n} \end{bmatrix} \begin{bmatrix} a_{n1} & \dots & a_{n,n-1} \end{bmatrix} \begin{bmatrix} E_1/1-\mathbf{ac} \\ \vdots \\ E_{n-1}/1-\mathbf{ac} \end{bmatrix} + \begin{bmatrix} E_1 \\ \vdots \\ E_{n-1} \end{bmatrix} \quad (\text{A1})$$

For each  $i$ ,  $i = 1, \dots, n-1$ , expression (A1) may be rewritten as:

$$X_i = \frac{[a_{in}a_{n1} \quad \dots \quad a_{in}a_{n,n-1}]}{1-\mathbf{ac}} \mathbf{E} + E_i \quad (\text{A1})'$$

The multiplier relationship for the  $i$ -th sector therefore takes the form:

$$X_i = \left( \frac{a_{in}\mathbf{a}}{1-\mathbf{ac}} \right) \mathbf{E} + E_i \quad (\text{A2})$$

Since  $\mathbf{E} = \xi \bar{X}_n \hat{\mathbf{c}}$  it follows that  $E_i = \xi \bar{X}_n a_{in}$ . By substituting these two expressions into (A2), and through some further manipulation, we have:

$$X_i = \left\{ a_{in} \left( \frac{\xi \mathbf{a} \hat{\mathbf{c}}}{1-\mathbf{ac}} \right) + \xi a_{in} \right\} \bar{X}_n \quad (\text{A2})'$$

The sectoral physical solution (A2)' corresponds to the effective sectoral production, which contrasts with the equilibrium full employment sectoral production, namely expression (7). Now it follows from (A2)', for the underdeveloped country  $U$ , that:

$$X_i^U = a_{in}^U \left( \frac{1}{1 - \sum_{i=1}^{n-1} a_{ni}^U a_{in}^U} \right) \left[ \sum_{j=1}^{n-1} a_{ni}^U \xi a_{in}^U \bar{X}_n \right] + \xi a_{in}^U \bar{X}_n \quad (\text{A3})$$

By isolating  $\bar{X}_n$  and rearranging terms one obtains obtain expression (14).

Below we provide proof of propositions 1 to 3.

### Proof of Proposition 1.

The proof is straightforward. If condition (5) holds then rearranging it we obtain:

$$\sum_{i=1}^{n-1} \xi a_{ni}^U a_{in}^U = 1 - \sum_{i=1}^{n-1} a_{ni}^U a_{in}^U. \text{ By substituting this result into the numerator of the right hand}$$

side of expression (12)', one obtains  $X_n = \bar{X}_n$ .  $\square$

### Proof of Proposition 2.

If condition (5) holds then from the last line of (7),  $X_n = \bar{X}_n$ . Besides, rearranging

expression (5) we obtain:  $\sum_{i=1}^{n-1} \xi a_{ni}^U a_{in}^U = 1 - \sum_{i=1}^{n-1} a_{ni}^U a_{in}^U$ . By substituting this result into the

numerator of the first term of the right hand side of expression (14), namely

$$X_i^U = \left( a_{in}^U \frac{\sum_{i=1}^{n-1} \xi a_{ni}^U a_{in}^U}{1 - \sum_{i=1}^{n-1} a_{ni}^U a_{in}^U} + \xi a_{in}^U \right) \bar{X}_n, \text{ it yields: } X_i^U = (a_{in}^U + \xi a_{in}^U) \bar{X}_n, \text{ which is the full}$$

employment equilibrium Pasinettian solution for the production of the  $i$ -th sector given

by the first  $n - 1$  lines of (7).  $\square$

### Proof of Proposition 3.

If  $\sum_{j=1}^{n-1} a_{ni}^U (a_{in}^U + \xi a_{i\hat{n}}^U) < 1$ , then it is possible to show after some algebraic manipulation

that:  $1 - \sum_{i=1}^{n-1} a_{ni}^U a_{in}^U > \sum_{i=1}^{n-1} \xi a_{ni}^U a_{i\hat{n}}^U$ . As a consequence,  $\frac{\sum_{i=1}^{n-1} \xi a_{ni}^U a_{i\hat{n}}^U}{1 - \sum_{i=1}^{n-1} a_{ni}^U a_{in}^U} < 1$ , and

$$X_i^U = \left( a_{in}^U \frac{\sum_{i=1}^{n-1} \xi a_{ni}^U a_{i\hat{n}}^U}{1 - \sum_{i=1}^{n-1} a_{ni}^U a_{in}^U} + \xi a_{i\hat{n}}^U \right) \bar{X}_n < (a_{in}^U + \xi a_{i\hat{n}}^U) \bar{X}_n. \text{ Now if } \sum_{i=1}^{n-1} a_{ni}^U (a_{in}^U + \xi a_{i\hat{n}}^U) > 1 \text{ then}$$

$\frac{\sum_{i=1}^{n-1} \xi a_{ni}^U a_{i\hat{n}}^U}{1 - \sum_{i=1}^{n-1} a_{ni}^U a_{in}^U} > 1$ . It follows that  $X_n > \bar{X}_n$ . In this case, solution (16)', namely the

sectoral Harrod foreign trade multiplier production, is higher than the corresponding sectoral equilibrium level of production.  $\square$